ON SOLVING A MULTI-CRITERIA DECISION MAKING PROBLEM USING FUZZY SOFT SETS IN SPORTS

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ABSTRACT
This paper deals with the problem of selecting a player for football team. The concept of fuzzy soft sets is applied to solve a multi-criteria decision making problem. Here, a new algorithm is proposed to select the best player. This algorithm gives a clear thought for a coach that will help to identify the best among the players.

Keywords: Fuzzy soft sets, Basic results of fuzzy soft sets, Trapezoidal fuzzy number, Mathematical modelling of the problem.

1. INTRODUCTION
In our daily life we often face some problems in which the right decision making is highly essential. But most of these cases we become confused about the right solution. To obtain the best solution of these problems, we have to consider various parameters relating to the solution. For this we can use fuzzy soft set theory. It was proposed by Molodtsov in 1999 [4] and studied the properties of fuzzy soft sets in the work of Maji et al. (2001) [5].

This paper is organized as follows: section 2 has definitions for better understanding the topic, section 3 deals with mathematical modelling of the problem and application, section 4 contains result of the problem, section 5 has conclusion.

2. PRELIMINARIES
2.1 DEFINITION
Let (F, S), (G, T) are the two soft sets over a common universe R, where S, T ⊆ F. Then,

- (F, S) is a sub soft set of (G, T) that means (F, S) ⊆ (G, T) if S ⊆ T and F (e) = G (e), for all e ∈ S.
- (F, S) = (G, T) if (F, S) ⊆ (G, T) and (G, T) ⊆ (F, S).
- The complement of a soft set (F, S), denoted by (F, S)′ = (F′, S), where F′ : S → 2^R such that F′ (e) = R ∼ F (e), for all e ∈ S.
- A soft set (F, S) is said to be a null soft set, if for all e ∈ S, F (e) = ∅, where ∅ is the null set of R.
- AND Operation of two soft sets (F, S) AND (G, T) is denoted by (H, S × T) = (F, S) ∧ (G, T), is defined as
  \[ H(\alpha, \beta) = F(\alpha) \cap G(\beta), \text{ for all } (\alpha, \beta) \in S \times T. \]
- Intersection of two soft sets (F, S) and (G, T) over the common universe R is the soft set (I, C) = (F, S) ∩ (G, T), where C = S ∩ T and I: C → 2^R. Such that, I (e) = F (e) or G (e), for all e ∈ C.
- OR Operation of two soft sets (F, S) OR (G, T) is denoted by (O, S × T) = (F, S) ∨ (G, T), is defined as
  \[ O(\alpha, \beta) = F(\alpha) \cup G(\beta), \text{ for all } (\alpha, \beta) \in S \times T. \]
- Union of two soft sets (F, S) and (G, T) over the common universe R is the soft set (U, C) = (F, S) ∪ (G, T), where C = S ∪ T and U: C → 2^R. Such that, for all e ∈ C,
2.2 FUZZY SOFT SET

Soft set theory is a generalization of fuzzy set theory. Let \( R \) be a Universal set, \( E \) be the set of parameters and \( S \subset E \). Also let \( I^R \) denote the set of all fuzzy subsets of \( R \). Then a pair \((F, S)\) is said to be a fuzzy soft set over \( R \), where \( F \) is a mapping from \( \gamma \) to \( I^R \).

The definitions of sub fuzzy soft set, null fuzzy soft set, intersection and union operations of fuzzy soft sets are similar to those definitions of sub soft set, null soft set, intersection and union operations of soft sets (crisp soft sets).

2.3 BASIC RESULTS OF SOFT SETS / FUZZY SOFT SETS

If \((F, S)\) and \((G, T)\) are the two soft sets / fuzzy soft sets. Then,

\[
\begin{align*}
(F, S) \cup (F, S) &= (F, S) \\
(F, S) \cap (F, S) &= (F, S) \\
(F, S) \cup \emptyset &= (F, S) \\
(F, S) \cap \emptyset &= (F, S) \\
((F, S) \cup (G, T))' &= (F, S)' \cup (G, T)' \\
((F, S) \cap (G, T))' &= (F, S)' \cap (G, T)'
\end{align*}
\]

2.4 TRAPEZOIDAL FUZZY NUMBER

\( \bar{\tilde{A}} \) is a normal fuzzy number represented by the quadruplet \((a, b, c, d)\) where \( a \leq b \leq c \leq d \) are real numbers and its membership function \( \mu_{\bar{\tilde{A}}}: X \rightarrow [0,1] \) is given below

\[
\mu_{\bar{\tilde{A}}}(x) = \begin{cases} 
\frac{x - a}{b - a}, & \text{if } a \leq x \leq b \\
1, & \text{if } b \leq x \leq c \\
\frac{x - d}{c - d}, & \text{if } c \leq x \leq d \\
0, & \text{if else}
\end{cases}
\]

2.5 ARITHMETIC OPERATIONS ON TRAPEZOIDAL FUZZY NUMBER

Let \( \bar{S} = (s_1, s_2, s_3, s_4) \) and \( \bar{T} = (t_1, t_2, t_3, t_4) \)

(a) **Addition** of the two trapezoidal numbers can be denoted as \( \bar{S} + \bar{T} \) and it is defined by

\[
\bar{S} + \bar{T} = (s_1 + t_1, s_2 + t_2, s_3 + t_3, s_4 + t_4)
\]

(b) **Subtraction** of the two trapezoidal numbers can be denoted as \( \bar{S} - \bar{T} \) and it is defined by

\[
\bar{S} - \bar{T} = (s_1 - t_1, s_2 - t_2, s_3 - t_3, s_4 - t_4)
\]

(c) **Multiplication** of the two trapezoidal numbers can be denoted as \( \bar{S} \times \bar{T} \) and it is defined by

\[
\bar{S} \times \bar{T} = (s_1 \times t_1, s_2 \times t_2, s_3 \times t_3, s_4 \times t_4)
\]

(d) **Complement of a trapezoidal fuzzy number** can be denoted by \( \bar{S}' \) and it is defined by

\[
\bar{S}' = (1 - s_4, 1 - s_3, 1 - s_2, 1 - s_1)
\]

(e) **Defuzzification** is the process of converting fuzzy numbers obtained from fuzzy inference into crisp values. For a trapezoidal number parameterized by \((s_1, s_2, s_3, s_4)\), then the defuzzification value \( t \) of the trapezoidal fuzzy number. It is calculated by

\[
t = \frac{s_4 + s_2 + s_3 + s_4}{4}
\]
3. APPLICATION OF FUZZY SOFT SET IN MULTI CRITERIA DECISION MAKING

3.1 MATHEMATICAL MODELLING OF THE PROBLEM

Suppose there are m alternatives (top players) \( T = \{T_1, T_2, ..., T_q\} \) and the selection committee members have taken some selection criteria as \( S = \{S_1, S_2, ..., S_r\} \) for preference evaluation of the top players. Their performance evaluation is expressed as fuzzy soft set \((F, S)\) over \( T \), where \( F: S \rightarrow I^T \), for each selection committee member.

From selection committee member 1 \((F_1)\)

\[
\begin{array}{cccc}
T_1 & T_2 & T_3 & ... & T_q \\
S_1 & a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & ... & a_{1q}^{(1)} \\
S_2 & a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & ... & a_{2q}^{(1)} \\
S_r & a_{r1}^{(1)} & a_{r2}^{(1)} & a_{r3}^{(1)} & ... & a_{rq}^{(1)} \\
\end{array}
\]

From selection committee member 2 \((F_2)\)

\[
\begin{array}{cccc}
T_1 & T_2 & T_3 & ... & T_q \\
S_1 & a_{11}^{(2)} & a_{12}^{(2)} & a_{13}^{(2)} & ... & a_{1q}^{(2)} \\
S_2 & a_{21}^{(2)} & a_{22}^{(2)} & a_{23}^{(2)} & ... & a_{2q}^{(2)} \\
S_r & a_{r1}^{(2)} & a_{r2}^{(2)} & a_{r3}^{(2)} & ... & a_{rq}^{(2)} \\
\end{array}
\]

And so on.

Then taking the average of all the above fuzzy soft sets we get the performance evaluation matrix as

\[
F(S) = \frac{1}{r} \left[ \begin{array}{cccc}
\bar{a}_{11} & \bar{a}_{12} & \bar{a}_{13} & ... & \bar{a}_{1q} \\
\bar{a}_{21} & \bar{a}_{22} & \bar{a}_{23} & ... & \bar{a}_{2q} \\
\bar{a}_{r1} & \bar{a}_{r2} & \bar{a}_{r3} & ... & \bar{a}_{rq} \\
\end{array} \right]
\]

Where \( \bar{a}_{ij} = \sum_{k=1}^{r} a_{ij}^{(k)}, i = 1,2, ..., q \text{ and } j = 1,2, ..., r \)

Suppose coacher is interested to select the best player on the basis of the recent selection committee member’s information (1), but he may have his own weightage to different selection criteria. For example preference weightage can be expressed as

\[
W = [w_1 \ w_2 \ w_3 \ ... \ w_r]
\]

Such that \( w_j \leq 1, j = 1,2, ..., r \). This restriction is to maintain the fuzzy property of the membership values.

Now to get the comprehensive decision matrix \( D \) for coacher, we multiply \((F(s))^T\) by the preference weightage matrix such that

\[
D = (\bar{a}_{ij} \times w_j)_{q \times r}
\]
Thus we get the comprehensive decision matrix as

\[
D = \begin{bmatrix}
  d_{11} & d_{12} & d_{13} & \ldots & d_{1r} \\
  d_{21} & d_{22} & d_{23} & \ldots & d_{2r} \\
  d_{31} & d_{32} & d_{33} & \ldots & d_{3r} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  d_{q1} & d_{q2} & d_{q3} & \ldots & d_{qr}
\end{bmatrix}
\]

Now alternative (Top players) weightage is calculated with normalization process

\[
\bar{W}_i = \tilde{T}_1 \odot (\tilde{T}_1 \oplus \tilde{T}_2 \oplus \ldots \oplus \tilde{T}_q)^{-1}
\]

Best Non-Fuzzy performance Value (BNP) is obtained from defuzzifying fuzzy alternative weights using centre of area (COA). For trapezoidal number COA is given as

\[
COA = \frac{S_1 + S_2 + S_3 + S_4}{4}
\]

Using COA, obtain weightage of the alternatives and using this, coacher can decide the best one for his foot-ball team among the top players.

Krishna Gogoi (2014) used an algorithm for solving fuzzy soft set theory in day to day problems. The algorithm was developed for solving multi-criteria decision making problems.

3.2 PROPOSED ALGORITHM

We shall follow the following algorithm for the solution of the problem discussed above.

- Input the performance evaluation of the top players by selection committee members as matrices.
- Find the average of the corresponding entries of all matrices.
- Multiply the weightage of the selection criteria of the coacher to the corresponding entries of each column to get the comprehensive decision matrix.
- Alternative weightage is calculated with normalization process.
- Defuzzify the fuzzy numbers for determining the degree of importance of alternatives.
- Obtain the weightage for each player and the player with maximum weightage is recommended as the best player.

Now, we illustrate the solution for the above discussed problem

Coacher is facing a problem for choosing the best player for his football team among the three top players, those players denoted by \(T_1, T_2, T_3\) respectively. He seeks advice from three selection committee members. The three selection committee members provided the information about the top players considering the parameters Technique & mind set, Game intelligence, Team player, physique which are denoted by \(S_1, S_2, S_3\) and \(S_4\) respectively. \(T = \{T_1, T_2, T_3\}\) be the set of three top players and \(S = \{S_1, S_2, S_3, S_4\}\) be the set of parameters. The information provided by the selection committee members forms the fuzzy soft sets \((F_1, S), (F_2, S)\) and \((F_3, S)\) over \(T\), where \(F_1, F_2, F_3\) are mappings from \(S\) into \(I^T\) given by the three selection committee members.

\[
F_1 (S_1) = \left\{ \begin{array}{ccc}
T_1 & T_2 & T_3 \\
0.5 & 0.8 & 0.7 \\
\end{array} \right\}
\]

\[
F_1 (S_2) = \left\{ \begin{array}{ccc}
T_1 & T_2 & T_3 \\
0.6 & 0.9 & 0.7 \\
\end{array} \right\}
\]

\[
F_1 (S_3) = \left\{ \begin{array}{ccc}
T_1 & T_2 & T_3 \\
0.6 & 0.7 & 0.8 \\
\end{array} \right\}
\]

\[
F_1 (S_4) = \left\{ \begin{array}{ccc}
T_1 & T_2 & T_3 \\
0.7 & 0.8 & 0.8 \\
\end{array} \right\}
\]

Then the fuzzy soft set \((F_3, S)\) is a parameterized family of all fuzzy sets over \(T\). Similarly, the soft sets \((F_2, S)\) and \((F_3, S)\), where

\[
F_2 (S_1) = \left\{ \begin{array}{ccc}
T_1 & T_2 & T_3 \\
0.6 & 0.9 & 0.8 \\
\end{array} \right\}
\]

\[
F_2 (S_2) = \left\{ \begin{array}{ccc}
T_1 & T_2 & T_3 \\
0.6 & 0.8 & 0.7 \\
\end{array} \right\}
\]
Now, the matrix representation of the above three fuzzy soft sets \((F_1, S), (F_2, S)\) and \((F_3, S)\) are

\[
F_2 (S_3) = \begin{bmatrix}
T_1 & T_2 & T_3 \\
0.5 & 0.7 & 0.7
\end{bmatrix}
\]

\[
F_2 (S_4) = \begin{bmatrix}
T_1 & T_2 & T_3 \\
0.7 & 0.8 & 0.8
\end{bmatrix}
\]

\[
F_3 (S_1) = \begin{bmatrix}
T_1 & T_2 & T_3 \\
0.4 & 0.7 & 0.6
\end{bmatrix}
\]

\[
F_3 (S_2) = \begin{bmatrix}
T_1 & T_2 & T_3 \\
0.5 & 0.7 & 0.6
\end{bmatrix}
\]

\[
F_3 (S_3) = \begin{bmatrix}
T_1 & T_2 & T_3 \\
0.6 & 0.8 & 0.7
\end{bmatrix}
\]

\[
F_3 (S_4) = \begin{bmatrix}
T_1 & T_2 & T_3 \\
0.6 & 0.7 & 0.8
\end{bmatrix}
\]

Now, the matrix representation of the above three fuzzy soft sets \((F_1, S), (F_2, S)\) and \((F_3, S)\) are

\[
(F_1, S) = \begin{bmatrix}
S_1 & T_1 & T_2 & T_3 \\
S_2 & 0.5 & 0.8 & 0.7 \\
S_3 & 0.6 & 0.9 & 0.7 \\
S_4 & 0.7 & 0.8 & 0.8
\end{bmatrix}
\]

\[
(F_2, S) = \begin{bmatrix}
S_1 & T_1 & T_2 & T_3 \\
S_2 & 0.6 & 0.9 & 0.8 \\
S_3 & 0.5 & 0.7 & 0.7 \\
S_4 & 0.7 & 0.8 & 0.8
\end{bmatrix}
\]

\[
(F_3, S) = \begin{bmatrix}
S_1 & T_1 & T_2 & T_3 \\
S_2 & 0.4 & 0.7 & 0.6 \\
S_3 & 0.5 & 0.7 & 0.6 \\
S_4 & 0.6 & 0.7 & 0.8
\end{bmatrix}
\]

Then taking the average of the above three fuzzy soft sets we get the performance evaluation matrix.

\[
F(S) = \begin{bmatrix}
S_1 & T_1 & T_2 & T_3 \\
S_2 & 0.5 & 0.8 & 0.7 \\
S_3 & 0.57 & 0.8 & 0.67 \\
S_4 & 0.57 & 0.73 & 0.73
\end{bmatrix}
\]

\[
F(S)^T = \begin{bmatrix}
T_1 & S_1 \\
T_2 & S_2 \\
T_3 & S_3 \\
T_4 & S_4
\end{bmatrix}
\]

Hence,

\[
F(S)^T = \begin{bmatrix}
T_1 & 0.5 & 0.57 & 0.57 & 0.67 \\
T_2 & 0.8 & 0.8 & 0.73 & 0.77 \\
T_3 & 0.7 & 0.67 & 0.73 & 0.8
\end{bmatrix}
\]

Next that the preference weightage of coacher to the different selection criteria is given by the matrix as

\[
W = \begin{bmatrix}
0.9 & 0.5 & 0.7 & 0.6
\end{bmatrix}
\]

Thus to get the comprehensive decision matrix \(D\) for coacher, we multiply \(F(S)^T\) by the preference weightage matrix and get \(D\) as follows
Alternative (Top players) weight calculation is shown in the table 1.

<table>
<thead>
<tr>
<th>Alternative weight, $W_i$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1$</td>
<td>0.2502</td>
<td>0.2793</td>
<td>0.281</td>
<td>0.295</td>
</tr>
<tr>
<td>$W_2$</td>
<td>0.400</td>
<td>0.392</td>
<td>0.359</td>
<td>0.352</td>
</tr>
<tr>
<td>$W_3$</td>
<td>0.350</td>
<td>0.328</td>
<td>0.359</td>
<td>0.352</td>
</tr>
</tbody>
</table>

Table 1

An important measure of top players is shown in the table 2.

<table>
<thead>
<tr>
<th>Top players</th>
<th>Weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>1.1055</td>
<td>3</td>
</tr>
<tr>
<td>$T_2$</td>
<td>1.503</td>
<td>1</td>
</tr>
<tr>
<td>$T_3$</td>
<td>1.389</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2

![Figure 1: Weightage of top players](image)

4. RESULT

From the important measures of top players, it is found top players weightage ranges in the order $C_2 > C_3 > C_1$ that shows player $C_2$ stands first followed by player $C_3$. Clearly, player $C_2$ is the best choice for coacher to his football team.

5. CONCLUSION

An application of fuzzy soft set theory is presented here in Multi-criteria decision making problem using multi-observer performance evaluation along with the performance weightage of the individual decision maker. From the comprehensive decision matrix, the result is derived. Thus, we can see that fuzzy soft set theory is useful for solving the day to day problems and it helps to take decision making in a critical situation. Here, fuzzy soft set is applied in developing a model for a football player selection.

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REFERENCES