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VERTEX ODD DIVISOR CORDIAL GRAPHS

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ABSTRACT

In this paper, we investigate the vertex odd divisor cordial labeling of $K_{2,n}$, S_n, $< K_{1,n}^{(1)}, K_{1,n}^{(2)} >$, Helm H_n, Flower Fl_n and switching of the apex vertex in Helm H_n.

Keywords : Vertex odd divisor cordial labeling, Vertex odd divisor cordial graph, divisor cordial labeling, divisor cordial graph.

1. Introduction

Throughout this paper we consider only finite, undirected and simple graphs. Let G be a graph with p vertices and q edges. For standard terminology and notations related to graph theory, we follow Harary [4], number theory, we refer to Burton [2] and graph labeling, and we refer to Gallian [3]. In [1], Cahit introduce the concept of cordial labeling of graph. In [13], Varatharajan et al. introduce the concept of divisor cordial labeling of graph. The divisor cordial labeling of various types of graph is presented in [5-9,11-14]. Motivated by the concept of divisor cordial labeling and odd labeling, we introduce a new special type of divisor cordial labeling called vertex odd divisor cordial labeling. Every divisor cordial graphs need not be vertex odd divisor cordial graphs. Also, every vertex odd divisor cordial graphs need not be divisor cordial graphs. In [10], Muthaiyan et al proved the wheel graph W_n, switching of a pendent vertex in path P_n, switching of a vertex in cycle C_n, Bistar $B_{n,n}$, $S(K_{1,n})$, $B_{n,n}^2$, $DS(B_{n,n})$, $S'(B_{n,n})$ and $< K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)} >$ are vertex odd divisor cordial graphs. The brief summaries of definition which are necessary for the present investigation are provided below.

Definition : 1.1
A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.
Definition : 1.2
A mapping f :V(G)→{0,1} is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. If for an edge e = uv, the induced edge labeling f* : E(G) → {0,1} is given by f*(e) = | f(u) − f(v) |. Then $v_f(i)$ = number of vertices of having label i under f and $e_f(i)$ = number of edges of having label i under f*. A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0)–v_f(1)| \le 1$ and $|e_f(0)–e_f(1)| \le 1$. A graph G is cordial if it admits cordial labeling.
Definition : 1.3
Let a and b be two integers. If a divides b means that there is a positive integer k such that b = ka. It is denoted by a | b. If a does not divide b, then we denote a ∤ b.
Definition : 1.4 [13]
Let G = (V(G), E(G)) be a simple graph and f : V(G) → {1,2,...,|V(G)|} be a bijection. For each edge uv, assign the label 1 if f(u) | f(v) or f(v) | f(u) and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) – e_f(1)| \le 1$. A graph with a divisor cordial labeling is called a divisor cordial graph.
Definition : 1.5
Let G = (V(G),E(G)) be a simple graph with n vertices and f : V → {1,3,…,2n–1} be a bijection. For each edge uv, assign the label 1 if either f(u) | f(v) or f(v) | f(u) and the label 0 otherwise. f is called a vertex odd divisor cordial labeling if $|e_f(0) − e_f(1)| \le 1$. A graph with vertex odd divisor cordial labeling is called a vertex odd divisor cordial graph.
Definition : 1.6
A wheel W_n, n ≥ 3, with n spokes is a graph that has a center vertex connected to all n vertices in cycle C_n.
Definition : 1.7
The shell S_n is the graph obtained by taking n−3 concurrent chords in cycle C_n. The vertex at which all the chords are concurrent is called the apex vertex.
Definition : 1.8
The Helm H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each rim vertex. It contains three types of vertices: an apex of degree n, n vertices of degree 4 and n pendant vertices.
Definition : 1.9
The flower Fl_n is the graph obtained from a Helm H_n by joining each pendant vertex to the apex of the Helm. It contains three types of vertices: an apex of degree 2n, n vertices of degree 4 and n vertices of degree 2.

2. Main Theorems
Theorem : 2.1
The graph $K_{2,n}$ is a vertex odd divisor cordial graph.
Proof.
Let G be a graph $K_{2,n}$.
Let u, v, $w_1,w_2, …,w_n$ be the vertices of G.
Then |V(G)| = n+2 and |E(G)| = 2n.
Define vertex labeling as f :V(G) → {1, 3, 5, …, 2n+3} as follows.
f(u) = 1,
f(v) = p, where p is the largest prime number such that p ≤ 2n+3.
Also, assign the remaining labels to the remaining vertices $w_1,w_2,…,w_n$ of G.
Since 1 divides any labels of the vertices adjacent to u also contribute n to $e_f(1)$ and p does not divide any labels of the vertices adjacent to v also contribute n to $e_f(0)$.
Then, $e_f(0)$ = n and $e_f(1)$ = n.
Therefore, $|e_f(0) − e_f(1)| = 0$.
Hence $K_{2,n}$ is a vertex odd divisor cordial graph.
Example : 2.1

The graph $K_{2,5}$ and its vertex odd divisor cordial labeling are shown in Figure 2.1.

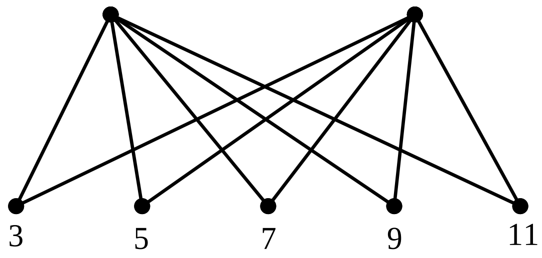

Figure 2.1

Theorem : 2.2

The graph S_n is a vertex odd divisor cordial graph for $n \geq 3$.

Proof.

Let G be a shell graph S_n. Let $v_1, v_2, ..., v_n$ be the vertices of G with v_1 as an apex vertex of G.

Then $|V(G)| = n$ and $|E(G)| = 2n-3$.

Define vertex labeling $f : V(G) \rightarrow \{1,3,5,\ldots,2n-1\}$ as follows.

$f(v_i) = 2i - 1$, for $1 \leq i \leq n$.

In view of the above labeling pattern we have, $e_g(0) = n-2$ and $e_g(1) = n-1$.

Therefore, $| e_f(0) - e_f(1) | \leq 1$.

Hence, S_n is a vertex odd divisor cordial graph for $n \geq 3$.

Example : 2.2

The graph S_7 and its vertex odd divisor cordial labeling are shown in Figure 2.2.

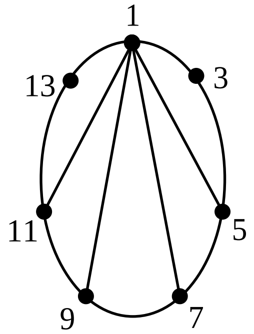

Figure 2.2

Theorem : 2.3

The graph $< K_{1,n}^{(1)}, K_{1,n}^{(2)} >$ is a vertex odd divisor cordial.

Proof.

Let G be a graph $< K_{1,n}^{(1)}, K_{1,n}^{(2)} >$.

Let $v_1, v_2, \ldots, v_n$ be the pendant vertices of $K_{1,n}^{(1)}$ and $u_1, u_2, \ldots, u_n$ be the pendant vertices of $K_{1,n}^{(2)}$, u and v be the apex vertices of $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$ respectively and they are adjacent to a new common vertex w.

Then $|V(G)| = 2n+3$ and $|E(G)| = 2n+2$.

Define vertex labeling as $f : V(G) \rightarrow \{1, 3, 5, \ldots, 4n+5\}$ as follows.

$f(u) = 1$,

$f(v) = p$, where p is the largest prime number such that $p \le 4n+5$.

Also, assign the remaining labels to the remaining vertices w, $v_1, v_2, \ldots, v_n$, $u_1, u_2, \ldots, u_n$ of G.

Since 1 divides any labels of the vertices adjacent to u also contribute n+1 to $e_f(1)$ and p does not divide any labels of the vertices adjacent to v also contribute n+1 to $e_f(0)$.

Therefore, $|e_f(0) - e_f(1)| = 0$.

Hence $< K_{1,n}^{(1)}, K_{1,n}^{(2)} >$ is vertex odd divisor cordial graph.

Example : 2.3

The graph $< K_{1,6}^{(1)}, K_{1,6}^{(2)} >$ and its vertex odd divisor cordial labeling are shown in Figure 2.3.

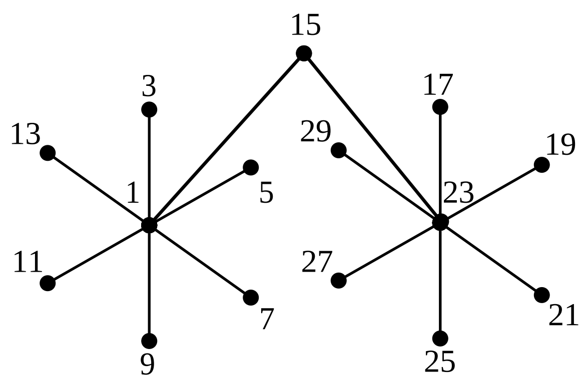

Figure 2.3

Theorem : 2.4

H_n is a vertex odd divisor cordial graph for $n \ge 3$.

Proof.

Let G be a Helm H_n.

Let v be the apex vertex, $v_1, v_2, \ldots, v_n$ be the vertices of degree 4 and $u_1, u_2, \ldots, u_n$ be the pendant vertices of H_n.

Then $|V(G)| = 2n+1$ and $|E(G)| = 3n$.

Define vertex labeling as $f : V(G) \to \{1, 3, 5, \ldots, 4n+1\}$ as follows.

$f(v) = 1$.

Case (i) : n = 3.

$f(v_i) = 1+2i$, for $1 \le i \le n$

$f(u_i) = 7+2i$, for $1 \le i \le n$.

Then $e_f(1) = 4$ and $e_f(0) = 5$.

Therefore, $| e_f(0) - e_f(1) | \le 1$.

Case (ii) : $n \ge 4$.

Consider $\left\lfloor \frac{n}{4} \right\rfloor = k$ and $\left\lfloor \frac{n}{2} \right\rfloor = m$.

$f(v_i) = 2k+2i-1$, for $1 \le i \le m$

$f(u_i) = 3f(v_i)$, for $1 \le i \le m$.

Also, assign the remaining labels to the remaining vertices, $v_{m+1}, v_{m+2}, \ldots, v_n$ and $u_{m+1}, u_{m+2}, \ldots, u_n$ such that $f(v_i) \nmid f(v_{i+1})$ where $m \le i \le n-1$, $f(v_n) \nmid f(v_1)$ and $f(v_i) \nmid f(u_i)$ where $m+1 \le i \le n$.

In view of above labeling pattern, we have $e_f(1) = \left\lfloor \frac{3n}{2} \right\rfloor$ and $e_f(0) = \left\lceil \frac{3n}{2} \right\rceil$

Therefore, $| e_f(0) - e_f(1) | \le 1$.

Hence, H_n is a vertex odd divisor cordial graph for $n \ge 3$.

Example : 2.4

The graph H_6 and its vertex odd divisor cordial labeling are shown in Figure 4.4.

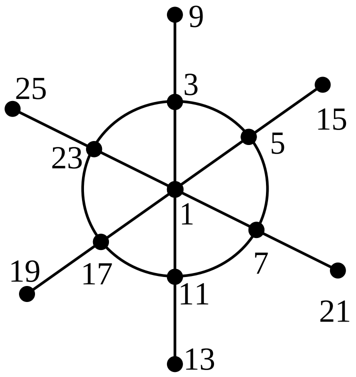

Figure 2.4

Theorem : 2.5

Fl_n is a vertex odd divisor cordial graph for $n \ge 3$.

Proof.

Let G be the graph Fl_n.

Let v be the apex vertex, $v_1, v_2, \ldots, v_n$ be the vertices of degree 4 and $u_1, u_2, \ldots, u_n$ be the vertices of degree 2 of G. Then $|V(G)| = 2n+1$ and $|E(G)| = 4n$.

Define vertex labeling $f : V(G) \rightarrow \{1,3,5,\ldots,4n+1\}$ as follows.

$f(v) = 1,$

Case (i) : $n \equiv 1 \pmod 3$

$f(v_i) = 3 + 4(i-1), \quad 1 \le i \le n-1$

$f(u_i) = f(v_i) + 2, \quad 1 \le i \le n-1.$

$f(v_n) = 4n+1$

$f(u_n) = 4n-1$

In view of the above labeling pattern, $e_f(0) = 2n = e_f(1)$.

Case (ii) : $n \equiv 0,3 \pmod 3$

$f(v_i) = 3 + 4(i-1), \quad 1 \le i \le n$

$f(u_i) = f(v_i) + 2, \quad 1 \le i \le n.$

In view of the above labeling pattern, $e_f(0) = 2n = e_f(1)$.

In both cases, we have $| e_f(0) - e_f(1) | \le 1$.

Hence, Fl_n is a vertex odd divisor cordial graph for $n \ge 3$.

Example : 2.5

The graph Fl_4 and its vertex odd divisor cordial labeling are shown in figure 2.5.

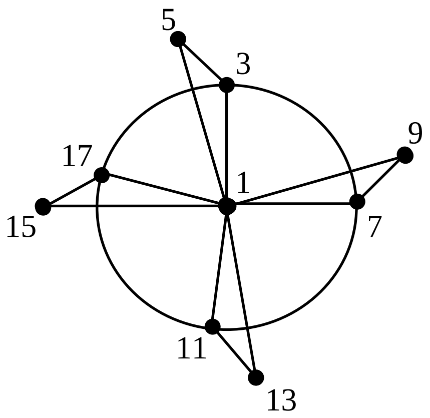

Figure 2.5

Theorem : 2.6

Switching of the apex vertex in Helm H_n is a vertex odd divisor cordial graph for $n \geq 3$.

Proof.

Let G be a Helm H_n. Let v be the apex vertex, $v_1, v_2, \ldots, v_n$ be the vertices of degree 4 and $u_1, u_2, \ldots, u_n$ be the pendant vertices of G.

Let G_v denotes graph obtained by switching of an apex vertex v of G. Then v become the apex vertex, $v_1, v_2, \ldots, v_n$ be the vertices of degree 3 and $u_1, u_2, \ldots, u_n$ be the vertices of degree 2 in G_v.

Here $|V(G_{v_1})| = 2n+1$ and $|E(G_{v_1})| = 3n$.

Define vertex labeling $f : V(G_{v_1}) \to \{1, 2, \ldots, 4n+3\}$ as follows:

$f(v) = 1$.

Case (i) : n = 3.

$f(v_i) = 1+2i$, for $1 \leq i \leq n$

$f(u_i) = 7+2i$, for $1 \leq i \leq n$.

Then $e_f(1) = 4$ and $e_f(0) = 5$.

Therefore, $| e_f(0) - e_f(1) | \leq 1$.

Case (ii) : $n \geq 4$.

Consider $\left\lfloor \frac{n}{4} \right\rfloor = k$ and $\left\lfloor \frac{n}{2} \right\rfloor = m$.

$f(v_i) = 2k+2i-1$, for $1 \leq i \leq m$

$f(u_i) = 3f(v_i)$, for $1 \leq i \leq m$.

Also, assign the remaining labels to the remaining vertices, $v_{m+1}, v_{m+2}, \ldots, v_n$ and $u_{m+1}, u_{m+2}, \ldots, u_n$ such that $f(v_i) \nmid f(v_{i+1})$ where $m \leq i \leq n-1$, $f(v_n) \nmid f(v_1)$ and $f(v_i) \nmid f(u_i)$ where $m+1 \leq i \leq n$.

In view of above labeling pattern, we have $e_f(1) = \left\lfloor \frac{3n}{2} \right\rfloor$ and $e_f(0) = \left\lceil \frac{3n}{2} \right\rceil$

Therefore, $| e_f(0) - e_f(1) | \leq 1$.

Hence, Switching of the apex vertex in helm H_n is a vertex odd divisor cordial graph for $n \geq 3$.

Example : 2.6

Switching of the apex vertex in helm H_6 and its vertex odd divisor cordial labeling are shown in Figure 2.6.

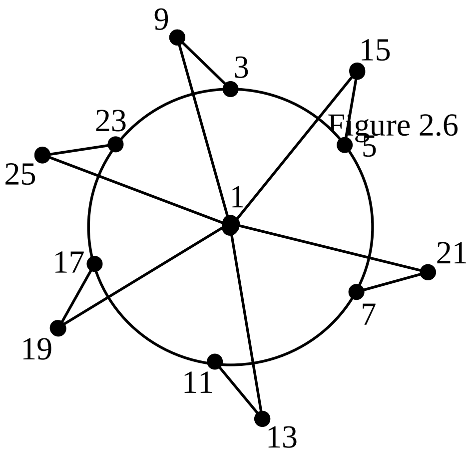

Figure 2.6

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