SUPER GEOMETRIC MEAN LABELING ON DOUBLE QUADRILATERAL SNAKE GRAPHS

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ABSTRACT

Let f: V(G) → {1,2,...,p+q} be an injective function for a vertex labeling “f” the induced edge labeling f* the induced edge labeling f*(e = uv) in defined by,

\[ f^*(e) = \sqrt{f(u)f(v)} \text{ or } \sqrt{f(u)f(v)}. \]

Then “f” is called a Super Geometric mean labeling if \[ \{ f(V(G)) \} \cup \{ f(e) : e \in E(G) \} = \{1,2,...,p+q\}. \] A graph which admits Super Geometric mean labeling is called Super Geometric mean graph.

In this paper, we investigate Super Geometric Mean Labeling On Double Quadrilateral snake Graphs.

Key words: Geometric mean graph, Super Geometric mean graph, Quadrilateral snake, Alternate Quadrilateral snake, Double Quadrilateral snake graph and Alternate Double Quadrilateral snake graph.
1. Introduction:
Throughout this paper we consider only finite, undirected, simple graphs. Let G be a graph with p vertices and q edges. For all terminologies and notations we follow Harary [2]. There are several types of labeling and a detailed survey can be found in Gallian [1].

The concept of “Geometric Mean Labeling” has introduced by S.Somasundaram, R. Ponraj and P. Vidhyarani in [6]. S. Somasundaram and R. Ponraj introduced “Mean Labeling” in [4].

The concept of “Harmonic mean labeling” has introduced by S. Somasundaram, R. Ponraj and S.S.Sandhya in [5]. C. Jeyasekaran, S.S.Sandhya and C.David Raj has introduced “Super Harmonic Mean Labeling” in [3].

In this paper we prove Super Geometric Mean Labeling on Double Quadrilateral snake Graphs.

The following definitions are necessary for the present study.

Definition 1.1: A graph \( G = (V,E) \) with p vertices and q edges is called a **Geometric Mean graph** if it is possible to label vertices \( x \in V \) with distinct label \( f(x) \) from 1,2,…,q+1 in such a way that when each edge \( e=uv \) is labeled with, 
\[ f(e=uv) = \left[ \sqrt{f(u)f(v)} \right] \] or 
\[ f(u)f(v) \] then the edge labels are distinct. In this case, “f” is called **Geometric Mean Labeling** of G.

Definition 1.2: Let \( f: V(G) \rightarrow \{1,2,...,p+q\} \) be an injective function. For a vertex labeling “f”, the induced edge labeling \( f^*(e=uv) \) is defined by, 
\[ f^*(e) = \left[ \sqrt{f(u)f(v)} \right] \] or 
\[ f(u)f(v) \]. Then “f” is called a **Super Geometric Mean Labeling** if \( \{f(V(G)) \cup \{ f(e) : e \in E(G) \} = \{1,2,...,p+q\} \). A graph which admits Super Geometric mean labeling is called **Super Geometric Mean Graph**.

Definition 1.3: The **Quadrilateral snake** \( Q_n \) is obtained from a Path \( u_1u_2…u_n \) by joining \( u_i, u_{i+1} \) to new vertices \( v_i, w_i \) respectively and then joining \( v_i \) and \( w_i \). That is every edge of Path is replaced by a Cycle \( C_4 \).

Definition 1.4: An **Alternate Quadrilateral snake** \( A(Q_n) \) is obtained from a Path \( u_1u_2…u_n \) by joining \( u_i, u_{i+1} \) (alternatively) to new vertices \( v_i, w_i \) respectively and then joining \( v_i \) and \( w_i \). That is every alternate edge of a Path is replaced by a Cycle \( C_4 \).

Definition 1.5: The **Double Quadrilateral snake** \( D(Q_n) \) consists of two Quadrilateral snakes that have a common Path.

Definition 1.6: The **Alternate Double Quadrilateral snake** \( A(D(Q_n)) \) consists of two Alternate Quadrilateral snakes that have a common Path.

We need the following theorems for providing our results.

**Theorem 1.7 [6]**: Triangular snakes and Quadrilateral snakes are Geometric mean graphs.

**Theorem 1.8 [5]**: Double Triangular and Alternate Double Triangular Snakes are Harmonic mean graphs.

**Theorem 1.9 [5]**: Double Quadrilateral and Alternate Double Quadrilateral snakes are Harmonic mean graphs.
Theorem 1.10: Double Triangular and Alternate Double Triangular snakes are Super Geometric mean graphs.

Theorem 1.11: Double Triangular and Alternate Triangular snakes are Geometric mean graphs.

Theorem 1.12: Double Quadrilateral and Alternate Double Quadrilateral snakes are Geometric mean graphs.

2. Main Results:

Theorem 2.1

Double Quadrilateral snakes $D(Q_n)$ are Super Geometric mean graphs.

Proof:

Let $P_n$ be the Path $u_1 u_2 \ldots u_n$

Join $u_i, u_{i+1}$ with four new vertices $v_i, w_i$ and $x_i, y_i, 1 \leq i \leq n-1$ by the edges $u_i v_i, u_{i+1} w_i, v_i w_i, u_i x_i, u_{i+1} y_i$ and $x_i y_i$

The labeling pattern is shown in the following figure.

![Diagram](image)

Figure: 1

Define a function $f: V(D(Q_n)) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_1) = 6$

$f(u_i) = 12i-11, 2 \leq i \leq n$

$f(v_1) = 9$

$f(v_i) = 12i-8, 2 \leq i \leq n-1$

$f(w_1) = 11$

$f(w_i) = 12i-3, 2 \leq i \leq n-1$

$f(x_1) = 4$

$f(x_i) = 12i-7, 2 \leq i \leq n-1$

$f(y_1) = 1$

$f(y_i) = 12i-1, 2 \leq i \leq n-1$

Edges are labeled with,

$f(u_1 u_2) = 8$

$f(u_i u_{i+1}) = 12i-5, 2 \leq i \leq n-1$

$f(v_1 u_i) = 7$

$f(v_i u_i) = 12i-10, 2 \leq i \leq n-1$

$f(w_1 u_2) = 12$
\[
\begin{align*}
  f(w_iu_{i+1}) &= 12i-2, \quad 2 \leq i \leq n-1 \\
  f(v_1w_i) &= 10 \\
  f(v_iw_i) &= 12i-6, \quad 2 \leq i \leq n-1 \\
  f(x_1u_1) &= 5 \\
  f(x_iu_i) &= 12i-9, \quad 2 \leq i \leq n-1 \\
  f(y_1u_2) &= 3 \\
  f(y_iu_{i+1}) &= 12i, \quad 2 \leq i \leq n-1 \\
  f(x_1y_i) &= 2 \\
  f(x_iy_i) &= 12i-4, \quad 2 \leq i \leq n-1
\end{align*}
\]

\[\therefore \text{We get distinct edge labels}\]
Hence \(\{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1,2,\ldots,p+q\}\).
\[\therefore D(Q_n) \text{ is a Super Geometric mean graph.}\]

**Example 2.2:** Super Geometric mean labeling of \(D(Q_5)\) is displaced below.

\[\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Figure 2}
\end{figure}\]

**Theorem 2.3**
Alternate Double Quadrilateral snakes \(A(D(Q_n))\) are Super Geometric mean graphs.

**Proof:**
Let \(G\) be the graph \(A(D(Q_n))\).
Let \(P_n\) be the Path \(u_1u_2\ldots u_n\).
Join \(u_i, u_{i+1}\) (alternatively) with four new vertices \(v_i, w_i, x_i\) and \(y_i\).
Here we consider two different cases.

**Case (1):** If the Alternate Double Quadrilateral snake \(A(D(Q_n))\) starts from \(u_1\), then we need to consider two subcases.
Subcase (1) (a): If $n$ is odd, then

Define a function $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,

- $f(u_1) = 6$
- $f(u_{2i-1}) = 14i-13, \quad 2 \leq i \leq \left(\frac{n-1}{2}\right)+1$
- $f(u_{2i}) = 14i-1, \quad 2 \leq i \leq \frac{n-1}{2}$
- $f(v_1) = 9$
- $f(v_i) = 14i-10, \quad 2 \leq i \leq \frac{n-1}{2}$
- $f(w_1) = 11$
- $f(w_i) = 14i-5, \quad 2 \leq i \leq \frac{n-1}{2}$
- $f(x_1) = 4$
- $f(x_i) = 14i-9, \quad 2 \leq i \leq \frac{n-1}{2}$
- $f(y_1) = 1$
- $f(y_i) = 14i-3, \quad 2 \leq i \leq \frac{n-1}{2}$

Edges are labeled with,

- $f(u_1 u_2) = 8$
- $f(u_i u_{i+1}) = 7i, \quad 2 \leq i \leq n-1$
- $f(u_1v_1) = 7$
- $f(u_{2i-1} v_i) = 14i-12, \quad 2 \leq i \leq \frac{n-1}{2}$
- $f(u_2 w_1) = 12$
- $f(u_{2i} w_i) = 14i-4, \quad 2 \leq i \leq \frac{n-1}{2}$
- $f(v_1 w_1) = 10$
- $f(v_1 w_i) = 14i-8, \quad 2 \leq i \leq \frac{n-1}{2}$
- $f(x_1 u_1) = 5$
- $f(x_i u_{2i-1}) = 14i-11, \quad 2 \leq i \leq \frac{n-1}{2}$
- $f(y_1 u_2) = 3$
- $f(y_i u_{2i}) = 14i-2, \quad 2 \leq i \leq \frac{n-1}{2}$
- $f(x_1 y_1) = 2$
- $f(x_i y_i) = 14i-6, \quad 2 \leq i \leq \frac{n-1}{2}$

The labeling pattern of $A(D(Q_4))$ is shown below
From the above labeling pattern, f provides a Super Geometric mean labeling of G.

**Subcase (1) (b):** If \( n \) is even, then

Define a function \( f: V(G) \rightarrow \{1,2,\ldots,p+q\} \) by,

\[
\begin{align*}
    f(u_1) &= 6 \\
    f(u_{2i-1}) &= 14i - 13, \quad 2 \leq i \leq \frac{n}{2} \\
    f(u_{2i}) &= 14i - 1, \quad 2 \leq i \leq \frac{n}{2} \\
    f(v_1) &= 9 \\
    f(v_i) &= 14i - 10, \quad 2 \leq i \leq \frac{n}{2} \\
    f(w_1) &= 11 \\
    f(w_i) &= 14i - 15, \quad 2 \leq i \leq \frac{n}{2} \\
    f(x_1) &= 4 \\
    f(x_i) &= 14i - 9, \quad 2 \leq i \leq \frac{n}{2} \\
    f(y_1) &= 1 \\
    f(y_i) &= 14i - 3, \quad 2 \leq i \leq \frac{n}{2}
\end{align*}
\]

Edges are labeled with,

\[
\begin{align*}
    f(u_1u_2) &= 8 \\
    f(u_iu_{i+1}) &= 7i, \quad 2 \leq i \leq n-1 \\
    f(u_1v_1) &= 7 \\
    f(u_{2i-1}v_i) &= 14i - 12, \quad 2 \leq i \leq \frac{n}{2} \\
    f(u_2w_1) &= 12 \\
    f(u_{2i}w_i) &= 14i - 4, \quad 2 \leq i \leq \frac{n}{2} \\
    f(v_1w_1) &= 10 \\
    f(v_iw_i) &= 14i - 8, \quad 2 \leq i \leq \frac{n}{2} \\
    f(x_1u_1) &= 5 \\
    f(x_iu_{2i-1}) &= 14i - 11, \quad 2 \leq i \leq \frac{n}{2} \\
    f(y_1u_2) &= 3 \\
    f(y_iu_{2i}) &= 14i - 2, \quad 2 \leq i \leq \frac{n}{2} \\
    f(x_1y_1) &= 2 \\
    f(x_iy_i) &= 14i - 6, \quad 2 \leq i \leq \frac{n}{2}
\end{align*}
\]
The labeling pattern of $A(D(Q_4))$ displaced below.

![Figure: 4](image)

In this case, $f$ provides a Super Geometric mean labeling of $G$.

**Case (2):** If the Alternate Double Quadrilateral snake $A(D(Q_n))$ starts from $u_2$, then we need to consider two subcases.

**Case (2) (a):** If $n$ is odd, then

Define a function $f: V(G) \to \{1,2,\ldots,p+q\}$ by,

- $f(u_{2i-1}) = 14i-13, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor +1$
- $f(u_{2i}) = 14i-11, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$
- $f(v_i) = 14i-8, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$
- $f(w_i) = 14i-3, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$
- $f(x_1) = 8$
- $f(x_i) = 14i-7, 2 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$
- $f(y_i) = 14i-1, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$

Edges are labeled with,

- $f(u_1u_2) = 2$
- $f(u_2u_3) = 7$
- $f(u_i u_{i+1}) = 7i-5, 3 \leq i \leq n-1$
- $f(u_{2i} x_i) = 14i-9, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$
- $f(u_{2i+1} y_i) = 14i, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$
- $f(x_i y_i) = 14i-4, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$
- $f(u_{2i} v_i) = 14i-10, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$
- $f(u_{2i+1} w_i) = 14i-2, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$
- $f(v_1w_1) = 9$
- $f(v_i w_i) = 14i-6, 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$

The labeling pattern of $A(D(Q_n))$ is given below.
From the above labeling pattern, we get distinct edge labels. Hence $f$ is a Super Geometric mean labeling of $G$.

**Subcase (2) (b):** If $n$ is even, then

Define a function $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by

- $f(u_1) = 14i-13, \ 1 \leq i \leq \frac{n}{2}$
- $f(u_2) = 14i-11, \ 1 \leq i \leq \frac{n}{2}$
- $f(v) = 14i-8, \ 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(w) = 14i-3, \ 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(x_1) = 8$
- $f(x_i) = 14i-7, \ 2 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(y_1) = 14i-1, \ 1 \leq i \leq \left(\frac{n-2}{2}\right)$

Edges are labeled with,

- $f(u_1u_2) = 2$
- $f(u_2u_3) = 7$
- $f(u_iu_{i+1}) = 7i-5, \ 3 \leq i \leq n-1$
- $f(u_2i, x_i) = 14i-9, \ 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(u_{2i+1}, y_i) = 14i, \ 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(x_i, y_i) = 14i-4, \ 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(u_2i, v_i) = 14i-10, \ 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(u_{2i+1}, w_i) = 14i-2, \ 1 \leq i \leq \left(\frac{n-2}{2}\right)$
- $f(v_1w_1) = 9$
- $f(v_iw_i) = 14i-6, \ 1 \leq i \leq \left(\frac{n-2}{2}\right)$

The labeling pattern of $A(D(Q_3))$ is shown below.
In this case $f$ provides a Super Geometric mean labeling of $G$.

From all the above cases, we conclude that Alternate Double Quadrilateral snakes are Super Geometric mean graphs.

References:


