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FACE AND TOTAL FACE SIGNED PRODUCT CORDIAL LABELING OF PLANAR GRAPHS

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ABSTRACT

In this paper, we investigate the face signed product cordial labeling of Pl_n, for n ≥ 5 and $Pl_{m,n}$, for m,n ≥ 3 and total face signed product cordial labeling of the Pl_n, n ≥ 4 and $Pl_{m,n}$, m, n ≥ 3.

Keywords : Signed product cordial labeling, Signed product cordial graph, Face Signed product cordial labeling, Total face signed product cordial labeling.

1. Introduction

By a graph, we mean a simple, finite, planar and undirected unless otherwise specified. A (p,q) planar graph G means a graph G = (V,E), where V is the set of vertices with |V| = p, E is the set of edges with |E| = q and F is the set of interior faces of G with |F| = number of interior faces of G, for terms not defined here, we refer to Harary [4]. For standard terminology and notations related to graph labeling, we refer to Gallian [3]. In [2], Cahit introduced the concept of cordial labeling of graph. The concept of signed product cordial labeling was introduced by Baskar Babujee et al.[1]. Sedlacek [7] defined a graph to be magic if it had an edge-labeling, with range the real numbers, such that the sum of the labels around any vertex equals some constant, independent of the choice of vertex. In 1983, Lih [5] introduced magic labelings of planar graphs where labels extended to faces as well as edges and vertices, an idea which he traced back to 13th century Chinese roots. Motivated by the concept of signed product cordial labeling and magic labeling, we introduce face signed product cordial labeling and total face signed product cordial labeling of graph. Ramanjaneyulu et al. proved cordial labeling of Pl_n and $Pl_{m,n}$ and total product cordial labeling of the $Pl_{m,n}$ under certain conditions in [6]. In this paper, we prove the face signed product cordial labeling of Pl_n, for n ≥ 5 and $Pl_{m,n}$, for m,n ≥ 3. Finally, we investigate the total face signed product cordial labeling of the Pl_n, n ≥ 4 and $Pl_{m,n}$, m, n ≥ 3. The brief summaries of definition which are necessary for the present investigation are provided below.

Definition : 1.1

The planar graph Pl_n = (V, E) has the vertex set V = $\{v_1, v_2, \ldots, v_n\}$, edge set E = $\{e, e_1, e_2, \ldots, e_{2n-4}, e'_1, e'_2, \ldots, e'_{n-3}\}$, where $e = (v_{n-1}, v_n)$, $e_{2i-1} = (v_i, v_{n-1})$ for $1 \le i \le n-2$, $e_{2i} = (v_i, v_n)$ for $1 \le i \le n-2$ and $e'_i = (v_i, v_{i+1})$ for $1 \le i \le n-3$ and interior face set F = $\{f, f_1, f_2, .., f_{2n-6}\}$, where $f = v_{n-1}v_nv_{n-2}v_{n-1}$, $f_{2i-1} = v_{n-1}v_iv_{i+1}v_{n-1}$ for $1 \le i \le n-3$ and $f_{2i} = v_n v_i v_{i+1} v_n$ for $1 \le i \le n-3$.

Definition : 1.2

The planar graph $Pl_{m,n}(V,E)$ has the vertex set $V = \{ v_1, v_2, ..., v_m, u_1, u_2, \ldots, u_n\}$, edge set $E = \{e_1,e_2,...,e_{2n},e'_1,e'_2,\ldots,e'_{2m-4}\}$, where $e_{2i-1} = (u_i,v_1)$ for $1 \le i \le n$, $e_{2i} = (u_i, v_2)$ for $1 \le i \le n$, $e'_{2i-5} = (u_1, v_{m+3-i})$ for $3 \le i \le m$ and $e'_{2i-4} = (v_{m+3-i}, u_2)$ for $3 \le i \le m$ and interior face set $F = \{f_1,f_2,...,f_{n-1},f'_1,f'_2,...,f'_{m-2}\}$, where $f_i = v_1u_iv_2u_{i+1}v_1$ for $1 \le i \le n-1$, $f'_1 = u_1v_1u_nv_3u_1$ and $f'_{i-1} = u_1v_iu_nv_{i+1}u_1$ for $3 \le i \le m-1$.

Definition : 1.3

A vertex labeling of graph G, $f : V(G) \to \{-1,1\}$ with induced edge labeling $f^*:E(G)\to\{-1,1\}$ defined by $f^*(uv) = f(u) f(v)$ is called a signed product cordial labeling if $|v_f(-1)-v_f(1)| \le 1$ and $|e_f(-1) - e_f(1)| \le 1$, where $v_f(-1)$ is the number of vertices labeled with -1, $v_f(1)$ is the number of vertices labeled with 1, $e_f(-1)$ is the number of edges labeled with -1 and $e_f(1)$ is the number of edges labeled with 1. A graph G is signed product cordial if it admits signed product cordial labeling.

We define face signed product cordial labeling and total face signed product cordial labeling as follows.

Definition : 1.4

For a planar graph G, the vertex labeling function is defined as $g : V(G) \to \{-1,1\}$ and $g(v)$ is called the label of the vertex v of G under g, induced edge labeling function $g^* : E(G) \to\{-1,1\}$ is given as if $e = uv$ then $g^*(e) = g(u) g(v)$ and induced face labeling function $g^{**} : F(G) \to \{-1,1\}$ is given as if $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_m$ are the vertices and edges of face f , then $g^{**}(f) = g(v_1) g(v_2) \ldots g(v_n) g^*(e_1) g^*(e_2) \ldots g^*(e_m)$. Let us denote $v_g(i)$ is the number of vertices of G having label i under g, $e_g(i)$ is the number of edges of G having label i under g^* and $f_g(i)$ is the number of interior faces of G having label i under g^{**} for $i = -1,1$. g is called face signed product cordial labeling of graph G if $|v_g(-1)-v_g(1)| \le 1$, $|e_g(-1)-e_g(1)| \le 1$ and $|f_g(-1)-f_g(1)| \le 1$.

A graph G is face signed product cordial if it admits face signed product cordial labeling.

Definition : 1.5

For a planar graph G, the vertex labeling function is defined as $g : V(G) \to \{-1,1\}$ and $g(v)$ is called the label of the vertex v of G under g, induced edge labeling function $g^* : E(G) \to\{-1,1\}$ is given as if $e = uv$ then $g^*(e) = g(u) g(v)$ and induced face labeling function $g^{**} : F(G) \to \{-1,1\}$ is given as if $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_m$ are the vertices and edges of face f, then $g^{**}(f) = g(v_1) g(v_2) \ldots g(v_n) g^*(e_1) g^*(e_2) \ldots g^*(e_m)$. Let $g(-1)$, $g(1)$ be the sum of the number of vertices, edges and interior faces having labels -1 and 1 respectively. g is called total face signed product cordial labeling of graph G if $|g(-1)-g(1)| \le 1$.

A graph G is total face signed product cordial if it admits total face signed product cordial labeling.

2. Main Theorems

Theorem : 2.1

The graph Pl_n is a face signed product cordial graph for $n \ge 5$.

Proof.

Let G be a planar graph Pl_n.

Let the vertex set be $V = \{v_1,v_2,\ldots,v_n\}$, edge set be $E=\{e,e_1,e_2,...,e_{2n-4},e'_1, e'_2,\ldots,e'_{n-3}\}$, where $e = (v_{n-1},v_n)$, $e_{2i-1} = (v_i,v_{n-1})$ for $1 \le i \le n-2$, $e_{2i} = (v_i, v_n)$ for $1 \le i \le n-2$ and $e'_i = (v_i, v_{i+1})$ for $1 \le i \le n-3$ and interior face set be $F = \{f,f_1,f_2,..,f_{2n-6}\}$, where $f = v_{n-1}v_nv_{n-2}v_{n-1}$, $f_{2i-1} = v_{n-1}v_iv_{i+1}v_{n-1}$ for $1 \le i \le n-3$ and $f_{2i} = v_n v_i v_{i+1} v_n$ for $1 \le i \le n-3$.

Then $|V(G)| = n$, $|E(G)| = 3n-6$ and $|F(G)| = 2n-5$.

Define vertex labeling $g : V(G) \to \{1, -1\}$ as follows.

$g(v_n) = 1$ and $g(v_{n-1}) = -1$,

Case (i) : $n \equiv 1 \pmod 4$

For $i = 1$ to $n - 2$

$g(v_i) = 1$, if $i \equiv 1,2 \pmod 4$

$g(v_i) = -1$, if $i \equiv 0,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1)+1 = \frac{n+1}{2}$, $e_g(-1) = e_g(1)+1 = \frac{3n-5}{2}$ and $f_g(1) = f_g(-1)+1 = n-2$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

Therefore, the graph Pl_n is a face signed product cordial graph for $n \equiv 1 \pmod 4$.

Case (ii) : $n \equiv 2 \pmod 4$

For $i = 1$ to $n - 2$

$g(v_i) = 1$, if $i \equiv 1,2 \pmod 4$

$g(v_i) = -1$, if $i \equiv 0,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) = \frac{n}{2}$, $e_g(1) = e_g(-1) = \frac{3n-6}{2}$ and $f_g(1) = f_g(-1)+1 = n-2$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

Therefore, the graph Pl_n is a face signed product cordial graph for $n \equiv 2 \pmod 4$.

Case (iii) : $n \equiv 3 \pmod 4$

For i = 1 to n – 2

$g(v_i) = 1$, if $i \equiv 1,2 \pmod 4$

$g(v_i) = -1$, if $i \equiv 0,3 \pmod 4$

In view of the above defined labeling pattern, we have

$v_g(1) = v_g(-1)+1 = \frac{n+1}{2}$, $e_g(-1) = e_g(1)+1 = \frac{3n-5}{2}$ and $f_g(-1) = f_g(1)+1 = n-2$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

Therefore, the graph Pl_n is a face signed product cordial graph for $n \equiv 3 \pmod 4$.

Case (iv) : $n \equiv 0 \pmod 4$

Subcase (i) : n = 4.

In order to satisfy the vertex condition for G, it is essential to assign label 1 and –1 to exactly 2 vertices. Any pattern assigning vertex labels satisfying vertex condition will induce edge labels for six number of edges in such a way that $|e_g(1)-e_g(-1)| \ge 2$, that is edge condition for G is violated. Thus the graph G under consideration is not a face signed product cordial graph when n = 4.

Subcase (ii) : n > 4.

$g(v_{n-3}) = -1$ and $g(v_{n-2}) = 1$,

For i = 1 to n – 4

$g(v_i) = 1$, if $i \equiv 1,2 \pmod 4$

$g(v_i) = -1$, if $i \equiv 0,3 \pmod 4$

In view of the above defined labeling pattern, we have

$v_g(1) = v_g(-1) = \frac{n}{2}$, $e_g(1) = e_g(-1) = \frac{3n-6}{2}$ and $f_g(-1) = f_g(1) +1 = n-2$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

Therefore, the graph Pl_n is a face signed product cordial graph for $n \equiv 0 \pmod 4$ and n > 4.

Hence, the graph Pl_n is face signed product cordial graph for $n \ge 5$.

Example : 2.1

Face signed product cordial labeling of Pl_5 and Pl_6 are shown in Figure 2.1(a) and Figure 2.1(b).

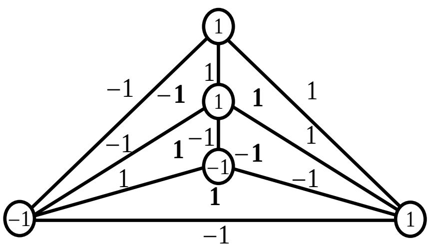

Figure 2.1(a)

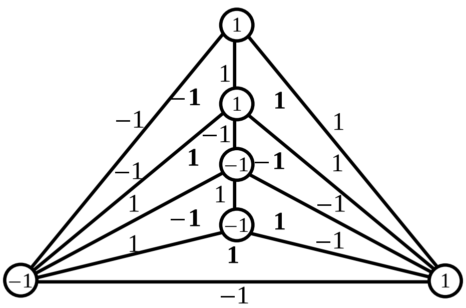

Figure 2.1(b)

Theorem 2.2

The graph $Pl_{m,n}$, m, n ≥ 3, is a face signed product cordial graph.

Proof.

Let G be a planar graph $Pl_{m,n}$.

Let the vertex set of G be $V = \{v_1,...,v_m,u_1,\ldots,u_n\}$, edge set of G be $E = \{e_1,e_2,...,e_{2n},e'_1,e'_2,\ldots,e'_{2m-4}\}$, where $e_{2i-1} = (u_i,v_1)$ for $1\le i \le n$, $e_{2i} = (u_i,v_2)$ for $1\le i \le n$, $e'_{2i-1} = (u_1,v_{i+2})$ for $1 \le i \le m-2$ and $e'_{2i} = (v_{i+2}, u_n)$ for $1 \le i \le m-2$ and interior face set of G be $F = \{f_1,f_2,...,f_{n-1},f'_1,f'_2,...,f'_{m-2}\}$, where $f_i = v_1u_iv_2u_{i+1}v_1$ for $1 \le i \le n-1$, $f'_1 = u_1v_1u_nv_3u_1$ and $f'_i = u_1v_{i+1}u_nv_{i+2}u_1$ for $2 \le i \le m-2$.

Then $|V(G)| = m+n$, $|E(G)| = 2m+2n-4$ and $|F(G)| = m+n-3$.

Define vertex labeling $g : V(G) \to \{1, -1\}$ as follows.

Case (1) : $n \equiv 0 \pmod 4$

$g(u_i) = 1$ if $i \equiv 1,2 \pmod 4$

$g(u_i) = -1$ if $i \equiv 0,3 \pmod 4$

subcase 1(i) : $m \equiv 0 \pmod 4$ $g(v_1) = -1$, $g(v_2) = 1$, $g(v_3) = 1$ and $g(v_4) = -1$,

For $5 \le i \le m$

$g(v_i) = 1$ if $i \equiv 1,2 \pmod 4$

$g(v_i) = -1$ if $i \equiv 0,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) = \frac{n+m}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) +1 = \frac{n+m-2}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 1(ii) : $m \equiv 1 \pmod 4$

$g(v_1) = -1$, $g(v_2) = 1$, $g(v_3) = 1$, $g(v_4) = -1$ and $g(v_5) = -1$,

For $6 \le i \le m$

$g(v_i) = 1$ if $i \equiv 2,3 \pmod 4$

$g(v_i) = -1$ if $i \equiv 0,1 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(-1) = v_g(1)+1 = \frac{n+m+1}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) = \frac{n+m-3}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 1(iii) : $m \equiv 2 \pmod 4$ $g(v_1) = -1$ and $g(v_2) = 1$,

For $3 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,3 \pmod 4$

$g(v_i) = -1$ if $i \equiv 1,2 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) = \frac{n+m}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(-1) = f_g(1) +1 = \frac{n+m-2}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 1(iv) : $m \equiv 3 \pmod 4$ $g(v_1) = -1$, $g(v_2) = 1$ and $g(v_3) = 1$,

For $4 \le i \le m$

$g(v_i) = -1$ if $i \equiv 0,1 \pmod 4$

$g(v_i) = 1$ if $i \equiv 2,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) +1 = \frac{n+m+1}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) = \frac{n+m-3}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

Case (2) : $n \equiv 1 \pmod 4$

For $1 \le i \le n-1$

$g(u_i) = 1$ if $i \equiv 1,2 \pmod 4$

$g(u_i) = -1$ if $i \equiv 0,3 \pmod 4$

subcase 2(i) : $m \equiv 0 \pmod 4$

$g(u_n) = 1$, $g(v_1) = -1$, $g(v_2) = 1$, $g(v_3) = -1$ and $g(v_4) = 1$,

For $5 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,3 \pmod 4$

$g(v_i) = -1$ if $i \equiv 1,2 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1)+1 = \frac{n+m+1}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(-1) = f_g(1) = \frac{n+m-3}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 2(ii) **:** $m \equiv 1 \pmod 4$

$g(u_n) = -1$, $g(v_1) = 1$, $g(v_2) = -1$, $g(v_3) = -1$, $g(v_4) = 1$ and $g(v_5) = 1$,

For $6 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,1 \pmod 4$

$g(v_i) = -1$ if $i \equiv 2,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) = \frac{n+m}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) +1 = \frac{n+m-2}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 2(iii) : $m \equiv 2 \pmod 4$ $g(u_n) = 1$, $g(v_1) = -1$ and $g(v_2) = 1$,

For $3 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,3 \pmod 4$

$g(v_i) = -1$ if $i \equiv 1,2 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1)+1 = \frac{n+m+1}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(-1) = f_g(1) = \frac{n+m-3}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 2(iv) : $m \equiv 3 \pmod 4$ $g(u_n) = -1$, $g(v_1) = -1$, $g(v_2) = 1$ and $g(v_3) = 1$,

For $4 \le i \le m$

$g(v_i) = 1$ if $i \equiv 2,3 \pmod 4$

$g(v_i) = -1$ if $i \equiv 0,1 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) = \frac{n+m}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(-1) = f_g(1) +1 = \frac{n+m-2}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

Case (3) : $n \equiv 2 \pmod 4$

$g(u_n) = -1$ and $g(u_{n-1}) = 1$

For $1 \le i \le n-2$

$g(u_i) = 1$ if $i \equiv 1,2 \pmod 4$

$g(u_i) = -1$ if $i \equiv 0,3 \pmod 4$

subcase 3(i) : $m \equiv 0 \pmod 4$ $g(v_1) = -1$, $g(v_2) = 1$, $g(v_3) = -1$ and $g(v_4) = 1$,

For $5 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,1 \pmod 4$

$g(v_i) = -1$ if $i \equiv 2,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) = \frac{n+m}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) +1 = \frac{n+m-2}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 3(ii) : $m \equiv 1 \pmod 4$

$g(v_1) = -1$, $g(v_2) = 1$, $g(v_3) = -1$, $g(v_4) = 1$ and $g(v_5) = 1$,

For $6 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,1 \pmod 4$

$g(v_i) = -1$ if $i \equiv 2,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1)+1 = \frac{n+m+1}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(-1) = f_g(1) = \frac{n+m-3}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 3(iii) : $m \equiv 2 \pmod 4$ $g(v_1) = -1$ and $g(v_2) = 1$,

For $3 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,1 \pmod 4$

$g(v_i) = -1$ if $i \equiv 2,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) = \frac{n+m}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) +1 = \frac{n+m-2}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 3(iv) : $m \equiv 3 \pmod 4$

$g(v_1) = -1$, $g(v_2) = 1$ and $g(v_3) = -1$,

For $4 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,1 \pmod 4$

$g(v_i) = -1$ if $i \equiv 2,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(-1) = v_g(1) +1 = \frac{n+m+1}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) = \frac{n+m-3}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

Case (4) : $n \equiv 3 \pmod 4$

$g(u_i) = 1$ if $i \equiv 1,2 \pmod 4$

$g(u_i) = -1$ if $i \equiv 0,3 \pmod 4$

subcase 4(i) : $m \equiv 0 \pmod 4$

$g(v_1) = -1$, $g(v_2) = 1$, $g(v_3) = -1$ and $g(v_4) = 1$,

For $5 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,1 \pmod 4$

$g(v_i) = -1$ if $i \equiv 2,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) +1 = \frac{n+m+1}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) = \frac{n+m-3}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 4(ii) : $m \equiv 1 \pmod 4$

$g(v_1) = -1$, $g(v_2) = 1$, $g(v_3) = 1$, $g(v_4) = -1$ and $g(v_5) = -1$,

For $6 \le i \le m$

$g(v_i) = 1$ if $i \equiv 2,3 \pmod 4$

$g(v_i) = -1$ if $i \equiv 0,1 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) = \frac{n+m}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) + 1 = \frac{n+m-2}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 4(iii) : $m \equiv 2 \pmod 4$ $g(v_1) = -1$ and $g(v_2) = 1$,

For $3 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,1 \pmod 4$

$g(v_i) = -1$ if $i \equiv 2,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) + 1 = \frac{n+m+1}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) = \frac{n+m-3}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

subcase 4(iv) : $m \equiv 3 \pmod 4$ $g(v_1) = -1$, $g(v_2) = 1$ and $g(v_3) = -1$,

For $4 \le i \le m$

$g(v_i) = 1$ if $i \equiv 0,1 \pmod 4$

$g(v_i) = -1$ if $i \equiv 2,3 \pmod 4$

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) = \frac{n+m}{2}$, $e_g(-1) = e_g(1) = n+m-2$ and $f_g(1) = f_g(-1) + 1 = \frac{n+m-2}{2}$.

Then $|v_g(-1) - v_g(1)| \le 1$, $|e_g(-1) - e_g(1)| \le 1$ and $|f_g(-1) - f_g(1)| \le 1$.

Therefore, the graph $Pl_{m,n}$, $m, n \ge 3$, is a face signed product cordial graph.

Illustration 2.2

Face signed product cordial labeling of $Pl_{4,4}$ and $Pl_{5,4}$ are shown in Figure 2.2(a) and Figure 2.2(b).

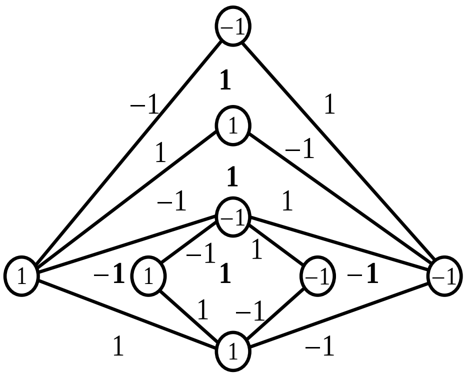

Figure 2.2(a)

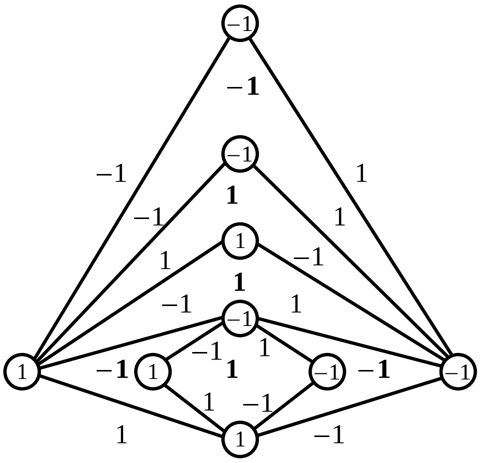

Figure 2.2(b)

Theorem 2.3

The graph Pl_n is a total face signed product cordial graph for $n \ge 3$.

Proof.

Let G be a planar graph Pl_n. Let the vertex set be $V = \{v_1, v_2, \ldots, v_n\}$, edge set be $E = \{e, e_1, \ldots, e_{2n-4}, e'_1, e'_2, \ldots, e'_{n-3}\}$, where $e = (v_{n-1}, v_n)$, $e_{2i-1} = (v_i, v_{n-1})$ for $1 \le i \le n-2$, $e_{2i} = (v_i, v_n)$ for $1 \le i \le n-2$ and $e'_i = (v_i, v_{i+1})$ for $1 \le i \le n-3$ and interior face set be $F = \{f, f_1, f_2, \ldots, f_{2n-6}\}$, where $f = v_{n-1}v_nv_{n-2}v_{n-1}$, $f_{2i-1} = v_{n-1}v_iv_{i+1}v_{n-1}$ for $1 \le i \le n-3$ and $f_{2i} = v_n\, v_i\, v_{i+1}\, v_n$ for $1 \le i \le n-3$. Then $|V(G)| = n$, $|E(G)| = 3n-6$ and $|F(G)| = 2n-5$.

Define vertex labeling $g : V(G) \to \{1, -1\}$ as follows.

$g(v_n) = 1$ and $g(v_{n-1}) = -1$,

For $i = 1$ to $n-2$

$g(v_i) = 1$, if $i \equiv 1,2 \bmod 4$

$g(v_i) = -1$, if $i \equiv 0,3 \bmod 4$

In view of the above defined labeling pattern, we have

(i) For $n \equiv 0,1,2 \pmod 4$, then $g(1) = g(-1) + 1 = 3n-5$

Thus, $|g(-1) - g(1)| \le 1$.

(ii) For $n \equiv 3 \pmod 4$, then $g(-1) = g(1) + 1 = 3n-5$

Thus, $|g(-1) - g(1)| \le 1$.

Therefore, the graph Pl_n is total face signed product cordial graph for $n \ge 3$.

Illustration 2.3

Total face signed product cordial labeling of Pl_4 is shown in Figure 2.3.

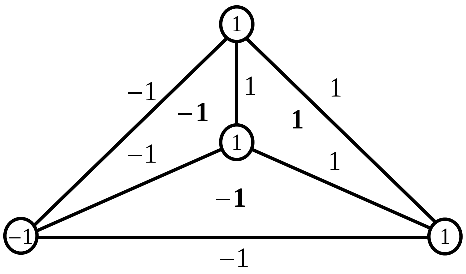

Figure 2.3

Theorem 2.4

The graph $Pl_{m,n}$, m, n ≥ 2, is a total face signed product cordial graph.

Proof.

Let G be a planar graph $Pl_{m,n}$. Let the vertex set of G be V = $\{v_1,...,v_m,u_1,\ldots, u_n\}$, edge set of G be E = $\{e_1,e_2,...,e_{2n},e'_1,e'_2,\ldots,e'_{2m-4}\}$, where $e_{2i-1} = (u_i,v_1)$ for $1 \le i \le n$, $e_{2i} = (u_i,v_2)$ for $1\le i \le n$, $e'_{2i-1} = (u_1,v_{i+2})$ for $1 \le i \le m-2$ and $e'_{2i} =(v_{i+2},u_n)$ for $1\le i \le m-2$ and interior face set of G be $F=\{f_1,f_2,...,f_{n-1},f'_1,f'_2,...,f'_{m-2}\}$, where $f_i = v_1u_iv_2u_{i+1}v_1$ for $1\le i \le n-1$, $f'_1 = u_1v_1u_nv_3u_1$ and $f'_i = u_1v_{i+1}u_nv_{i+2}u_1$, $2\le i \le m-2$.

Then $|V(G)| = m+n$, $|E(G)| = 2m+2n-4$ and $|F(G)| = m+n-3$.

Define vertex labeling $g : V(G) \to \{1, -1\}$ as follows.

$g(u_1) = -1$, $g(v_1) = 1$ and $g(v_2) = -1$,
$g(u_i) = 1$, for $i = 2,3,\ldots,n$.
$g(v_i) = 1$, for $i = 3,4,\ldots,m$.

In view of the above defined labeling pattern, we have $g(1) = g(-1)+1 = 2(m+n)-3$. Thus, $|\, g(-1) - g(1)| \le 1$.

Therefore, the graph $Pl_{m,n}$ is total face signed product cordial graph for m, n ≥ 2.

Illustration 2.4

Total face signed product cordial labeling of $Pl_{5,4}$ is shown in Figure 2.4.

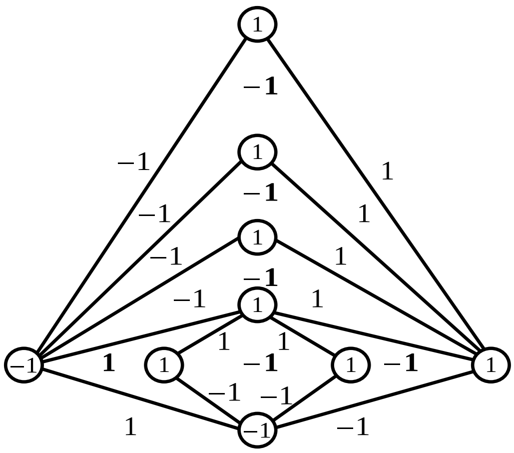

Figure 3.4

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