ON SUPER GEOMETRIC MEAN LABELING OF GRAPHS

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ABSTRACT

Let G be a graph with p vertices and q edges. Let \( f : V(G) \to \{1, 2, 3, \ldots, p+q\} \) be a injective function. For a vertex labeling \( f \), the induced edge labeling \( f(e=uv) \) is defined by \( f(e) = \sqrt{f(u)f(v)} \) (or) \( \sqrt{f(u)f(v)} \). Then \( f \) is called a Super Geometric mean labeling if \( f(V(G)) \cup \{f(e) / e \in E(G)\} = \{1, 2, 3, \ldots, p+q\} \). A graph which admits Super Geometric mean labeling is called Super Geometric mean graph. In this paper, we investigate Super geometric mean labeling of some graphs.

Keywords: Graph, Super Geometric mean labeling, Super Geometric mean graph, Comb, Triangular snake, Dragon.

1. INTRODUCTION

We begin with simple, finite, connected and undirected graph \( G \) with \( p \) vertices and \( q \) edges. For a detailed survey of graph labeling we refer to Gallian [1]. Terms are not defined here are used in the sense of Harary [2]. S. Somasundaram and R. Ponraj introduced mean labeling of graphs in [3], [4]. R.Ponraj and D. Ramya introduced Super mean labeling of graphs in [5]. S. Somasundram, P. Vidhyarani and R. Ponraj introduced Geometric mean labeling of graphs in [6]. In this paper, we investigate Super Geometric mean labeling of some graphs. We now give the following definitions which are useful for the present investigation.

**Definition 1.1.** Let \( f : V(G) \to \{1, 2, 3, \ldots, p+q\} \) be a injective function. For a vertex labeling \( f \), the induced edge labeling \( f(e=uv) \) is defined by \( f(e) = \sqrt{f(u)f(v)} \) (or) \( \sqrt{f(u)f(v)} \). Then \( f \) is called
a Super Geometric mean labeling if $f(V(G)) \cup \{f(e) \mid e \in E(G)\} = \{1, 2, 3, \ldots, p + q\}$. A graph which admits Super Geometric mean labeling is called Super Geometric mean graph.

**Definition 1.2.** The Corona $G_1 \circ G_2$ of two graphs $G_1$ and $G_2$ is defined as the graph $g$ obtained by taking one copy of $G_1$ (which has $p_1$ vertices) and $p_1$ copies of $G_2$ and then joining the $i^{th}$ vertex of $G_1$ to every vertex in the $i^{th}$ copy of $G_2$.

**Definition 1.3.** The graph $P_n \square K_1$ is called Comb.

**Definition 1.4.** The Graph $C_m @ P_n$ is called Dragon.

**II. MAIN RESULTS**

**Theorem 2.1**

The path $P_n$ is Super Geometric mean graph for any $n \geq 2$.

**Proof:**

Let the vertices of $P_n$, \{ $v_i : 1 \leq i \leq n$\} and the edges of $P_n$, \{ $e_i = (v_i, v_{i+1}) : 1 \leq i \leq n-1$ \}

![Fig. 1 P_n with ordinary labeling](image)

We now label the vertices of $P_n$,

Define a function $f : V(P_n) \rightarrow \{1, 2, \ldots, p+q\}$

$$f(v_i) = 2i - 1, \quad 1 \leq i \leq n$$

Then the induced edge labels are

$$f(e_i) = 2i, \quad 1 \leq i \leq n-1$$

Thus both vertices and edges together get distinct labels from \{1, 2, \ldots, p+q\}

Hence $P_n$ is a Super Geometric mean graph for any $n \geq 2$.

**Example 2.2**

![Example 2.2 graph](image)
Theorem 2.3

The Cycle $C_n$ is a Super Geometric mean graph for any $n \geq 3$.

Proof:

Let the vertices of $C_n$, \{ $v_i : 1 \leq i \leq n$\} and the edges of $C_n$,\{ $e_i : 1 \leq i \leq n$\} as represented in Fig.3

$$f(v_i) = \begin{cases} 1 & i=1 \\ 4i-2 & 2 \leq i \leq \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ & \& 2 \leq i \leq \frac{n}{2}, \text{ if } n \text{ is even} \\ 2n & i=\frac{n+1}{2}, \text{ if } n \text{ is odd} \\ & \& i=\frac{n+2}{2}, \text{ if } n \text{ is even} \\ 4(n-i+1) & \frac{n+3}{2} \leq i \leq n, \text{ if } n \text{ is odd} \\ & \& \frac{n+4}{2} \leq i \leq n, \text{ if } n \text{ is even} \end{cases}$$
Then the induced edge labels are

\[
f(e_i) = \begin{cases} 
4i - 1 & \text{if } 1 \leq i \leq \frac{n}{2}, \text{ if } n \text{ is even} \\
& \& 1 \leq i \leq \frac{n-1}{2}, \text{ if } n \text{ is odd} \\
f\left(\frac{e_n}{2}\right) + 2(n - 2i + 1) & \frac{n+2}{2} \leq i \leq n-1, \text{ if } n \text{ is even} \\
& f\left(\frac{e_{n+1}}{2}\right) + 2(n - 2i + 2) & \frac{n+1}{2} \leq i \leq n-1, \text{ if } n \text{ is odd} \\
2 & i = n 
\end{cases}
\]

Thus both vertices and edges together get distinct labels from \(\{1, 2, \ldots, p+q\}\).

Hence \(C_n\) is a Super Geometric mean graph for any \(n \geq 3\).

**Example 2.4**

![Graph](image)

Fig. 4 Super Geometric Mean labeling of \(C_6\).

**Theorem 2.5**

The Comb \(P_n \Box K_1, (n \geq 2)\) is a Super Geometric mean graph.

**Proof:**

Let \(\{v_i : 1 \leq i \leq n\}\) and \(\{u_i : 1 \leq i \leq n\}\) be the vertices and \(\{e_i = u_i u_{i+1} : 1 \leq i \leq n-1\}\) and \(\{e'_i = u_i v_i : 1 \leq i \leq n\}\) represented in fig.5
We now label the vertices of $P_n \Box K_1$.

Define a function $f : V(P_n \Box K_1) \rightarrow \{1, 2, \ldots, p+q \}$

$$f(u_i) = \begin{cases} 4i-1, & \text{if } i \text{ is odd} \\ 4i-3, & \text{if } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 4i-3, & \text{if } i \text{ is odd} \\ 4i-1, & \text{if } i \text{ is even} \end{cases}$$

Then the induced edge labels are

$$f(e_i) = 4i, \quad 1 \leq i \leq n-1$$
$$f(e'_i) = 4i-2, \quad 1 \leq i \leq n$$

Thus both vertices and edges together get distinct labels from $\{1, 2, \ldots, p+q \}$

Hence, $P_n \Box K_1$ is a Super Geometric mean graph.

**Example 2.6**

![Graph](image.png)

Fig. 6 Super Geometric mean labeling $P_n \Box K_1$. 
Theorem 2.7

The Dragon’s $C_m \oplus P_n$ are a Super Geometric mean graph.

Proof:

Let $\{v_i : 1 \leq i \leq m\}$ be the vertices of $C_m$ and $\{u_i : 1 \leq i \leq n\}$ be the vertices of $P_n$ and $\{e_i : 1 \leq i \leq m+n-1\}$ be the edges of $C_m \oplus P_n$.

Identify $\frac{v_{m+1}}{2} = u_i$ if $n$ is odd

Identify $\frac{v_{m+2}}{2} = u_i$ if $n$ is even

We first, label the vertices of $C_m \oplus P_n$ as follows,

Define $f : V (C_m \oplus P_n) \rightarrow \{1, 2, \ldots, p+q\}$ by

$$f(v_i) = \begin{cases} 1 & i = 1 \\ 4i - 2 & 2 \leq i \leq \frac{m-1}{2}, \text{ if } m \text{ is odd} \\ & \& 2 \leq i \leq \frac{m}{2}, \text{ if } m \text{ is even} \\ 2m & i = \frac{m+1}{2}, \text{ if } m \text{ is odd} \\ & \& i = \frac{m+2}{2}, \text{ if } m \text{ is even} \\ 4(m-i+1) & \frac{m+3}{2} \leq i \leq m, \text{ if } m \text{ is odd} \\ & \& \frac{m+4}{2} \leq i \leq m, \text{ if } m \text{ is even} \\ \end{cases}$$
f(u_i) = 2(m + i -1), \quad 1 \leq i \leq n

Then the induced edge labels are

\[
f(e_i) = \begin{cases} 
4i - 1 & \text{if } i \leq \frac{m}{2}, \text{ if } m \text{ is even} \\
\frac{m+2}{2} - i \leq m-1, \text{ if } m \text{ is even} \\
\frac{m+1}{2} - i \leq m-1, \text{ if } m \text{ is odd} \\
i = m
\end{cases}
\]

Thus both vertices and edges together get distinct labels from \{1, 2, \ldots, p+q\}.

Hence, the Dragon’s \(C_m @ P_n\) are a Super Geometric mean graph.

**Example 2.8**

![Diagram of Super Geometric mean labeling of \(C_8 @ P_4\).]

**Theorem 2.9**

The Triangular snake \(T_n\) are a Super Geometric mean graph.

**Proof:**

Let the vertices of \(T_n\) be \{ \(v_i : 1 \leq i \leq n\) \} and \{ \(u_i : 1 \leq i \leq n-1\) \}. 


Let the edges of $T_n$ be $\{e_i : 1 \leq i \leq n-1\}$ and $\{e_i' : 1 \leq i \leq 2(n-1)\}$.

![Diagram of $T_n$ with ordinary labeling]

Now we label the vertices of $T_n$ as follows,

Define $f : V(T_n) \rightarrow \{1, 2, \ldots, p+q\}$ by

$$
\begin{align*}
  f(v_i) &= 5i - 4, & 1 \leq i \leq n \\
  f(u_i) &= 5i - 1, & 1 \leq i \leq n-1
\end{align*}
$$

Then the induced Edge labels are

$$
\begin{align*}
  f(e_i) &= \begin{cases} 
    5i - 2, & 1 \leq i \leq n-1 \\
    \frac{5i}{2}, & \text{if } i \text{ is even}
  \end{cases} \\
  f(e_i') &= \frac{5i}{2}, & \text{if } i \text{ is even}
\end{align*}
$$

Thus both the vertices and edges together get distinct labels from $\{1, 2, \ldots, p+q\}$.

Hence, the Triangular snake $T_n$ are a Super Geometric mean graph.

**Example 2.10**

![Diagram of Super Geometric mean labeling of $T_4$.]
REFERENCES


